ABERRATIONS

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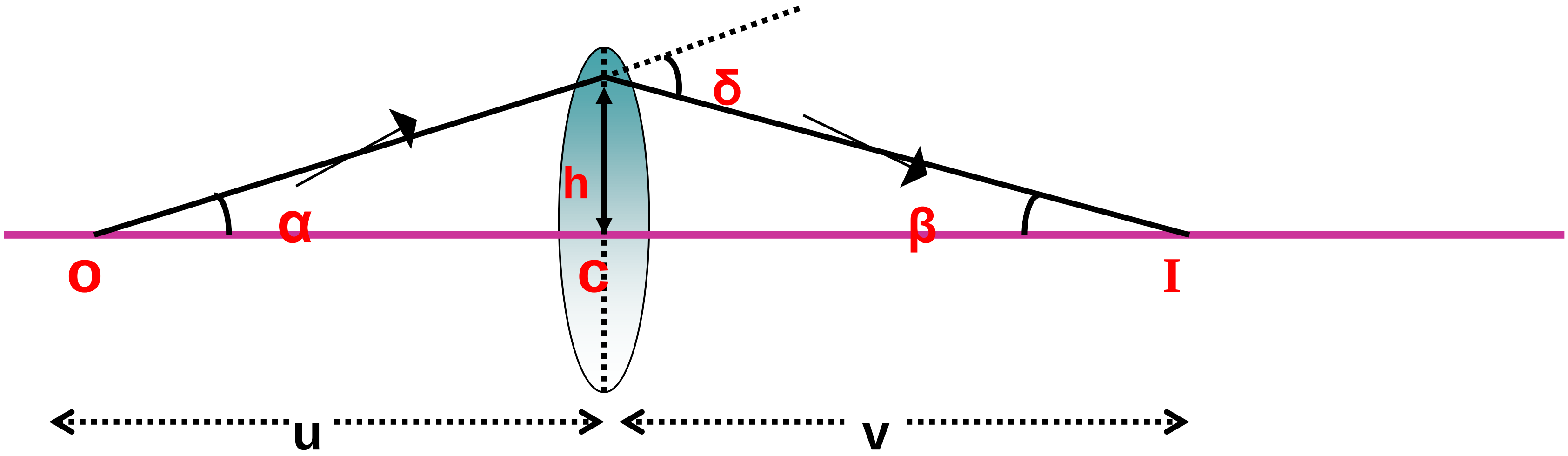
Deviation produced by a thin lens



From the ▲AIO, $\delta = \alpha + \beta$

Hence $\alpha = \tan\alpha$ and $\beta = \tan\beta$

$\alpha = h/u$ and $\beta = h/v$

Now, $\delta = h/u + h/v$ since u=-ve and v=+ve

$\delta = h/v - h/u = h(1/v - 1/u)$

Or $\delta = h/f$

ABERRATIONS

- Definition of Aberrations.
- Types of Aberrations.
  - Chromatic Aberrations.
  - Monochromatic Aberrations.
- Mathematical Analysis.
- Methods of Elimination.

What is an Aberration?

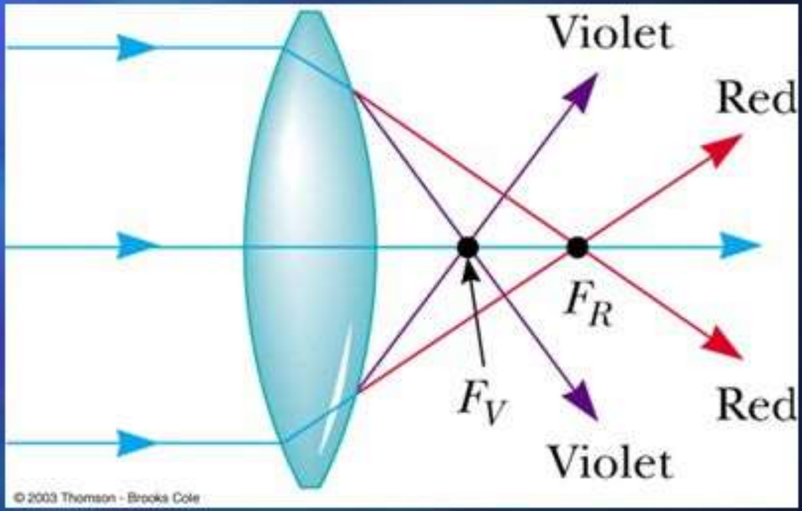
The deviations or defects observed in the actual size, position, shape and colour in the image of an object formed by a single lens are called Aberrations.

CAUSE FOR ABERRATIONS ?

- **These aberrations are caused due to the convergence of different rays to different points.**
- **Formation of Multicoloured image.**
- **Disability of lens to converge different colours at different points.**

Chromatic Aberration

- Different wavelengths of light refracted by by a lens focus at different points
  - Violet rays are refracted more than red rays
  - The focal length for red light is greater than the focal length for violet light
- Chromatic aberration can be minimized by the use of a combination of converging and diverging lenses



TYPES OF CHROMATIC ABERRATIONS

- **Longitudinal Chromatic Aberrations.**

  The formation of images of different colours in different positions along the axis

- **Lateral /Transverse Chromatic Aberrations.**

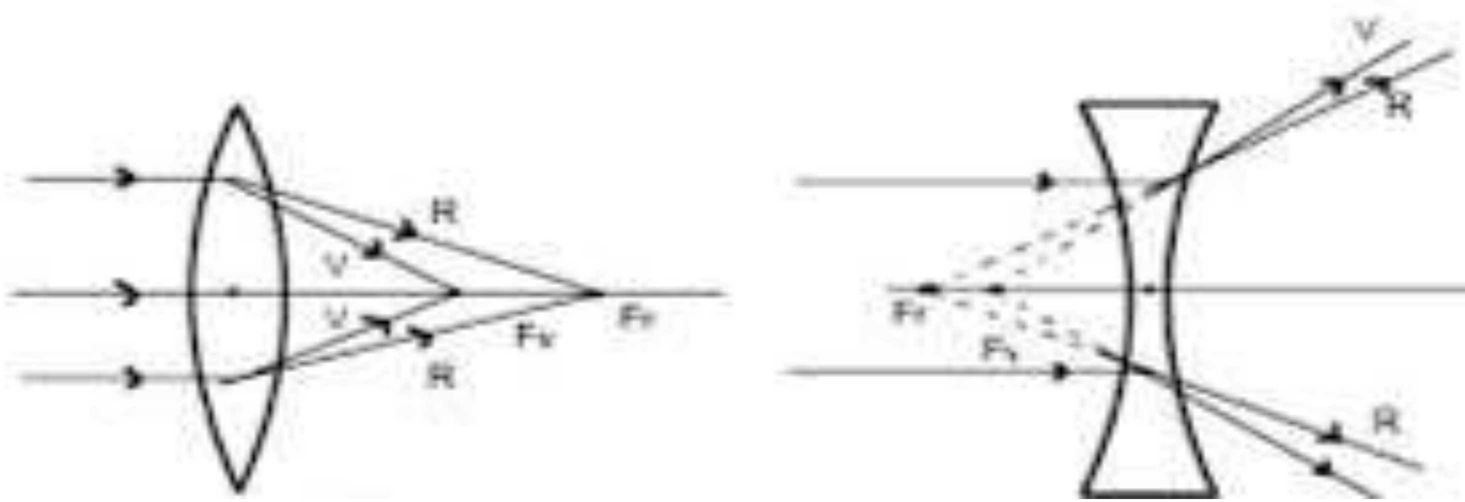
  The defect in images of different colours are formed of different sizes perpendicular to the axis .

Visuals of Chromatic Aberrations

LONGITUDINAL CHROMATIC ABERRATIONS(L.C.A)

If the object is placed at infinity longitudinal chromatic aberration will be equal to the difference in focal lengths for red and violet rays.The difference (f_r — f_v) measures axial or longitudinal chromatic aberration.

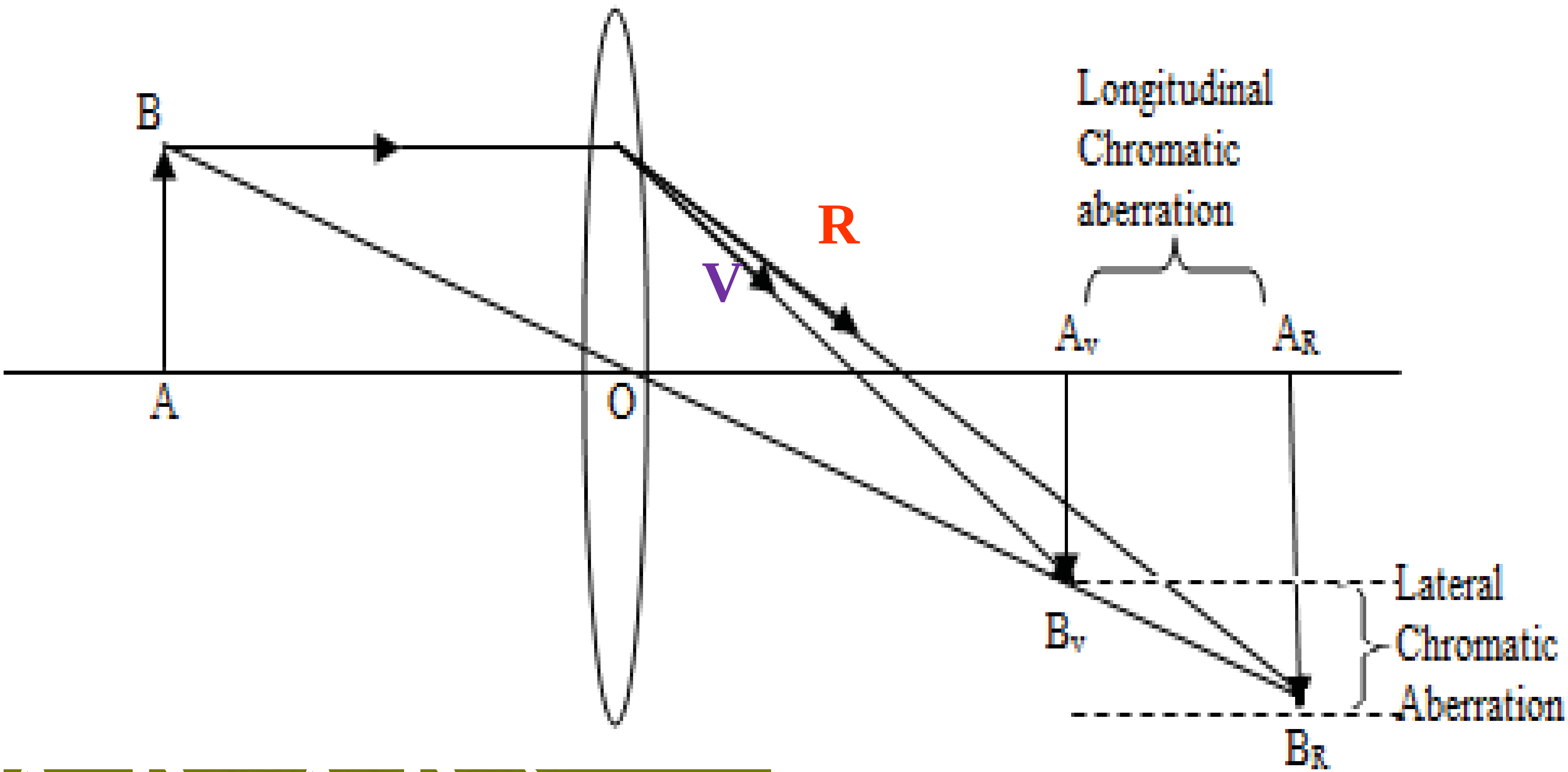
White Light =>



for Convex Lens (f_r - f_v) is "+" ve,
for Concave Lens (f_r - f_v) is " --" ve

L.C.A.= f_r - f_v

Lateral/Transverse Chromatic Aberration



Lateral Chromatic Aberration.

or Longitudinal chromatic

CALCULATION OF LONGITUDINAL CHROMATIC ABERRATION OF A THIN LENS (1)When the object is situated at infinity.

General formulae $1/f=(\mu-1)(1/R_1-1/R_2)$,

If f_v,f_R and f_Y be the focal lengths of the lens for violet, red and yellow colours.

$1/f_v=(\mu_v-1)(1/R_1-1/R_2)$, ………..(1)

$1/f_R=(\mu_R-1)(1/R_1-1/R_2)$,……………(2)

$1/f_Y=(\mu_Y-1)(1/R_1-1/R_2)$……………….(3)

Equation (2) - (1) => $1/f_V-1/f_R=(\mu_V-\mu_R)(1/R_1-1/R_2)$,

$(f_R-f_v)/ f_v.f_R = \{(\mu_V-\mu_R)/ (\mu_Y-1)\} (\mu_Y-1)(1/R_1-1/R_2)$,

Here $\{(\mu_V-\mu_R)/ (\mu_Y-1)\}=\omega$, $f_V.f_R = f_Y^2$, $(\mu_Y-1)(1/R_1-1/R_2)= 1/f_Y$

- **CALCULATION OF LONGITUDINAL CHROMATIC ABERRATION OF A THIN LENS (2)WHEN THE OBJECT IS AT A FINITE DISTANCE.**

  - **Suppose for a thin lens object distance "u", image distance "v", focal length "f" then the relation between above three quantities is**

- **1/v-1/u = 1/f** ....(1)

- for single object(length "u") voilet colour image distance "v_V" and
- red colour image distance "v_R" then

- **$1/v_V - 1/u = 1/f_V$** ....(2) , **$1/v_R - 1/u = 1/f_R$** ....(3)

- Subtract eqn (2) - (3) => **$(1/v_V - 1/u)$ -- $(1/v_R - 1/u) = 1/f_V$ -- $1/f_R$**

- simplifying **$V_R - V_V = \acute{\omega}.V^2/f$**............(4) this equation represent Longitudinal Chromatic Aberration WHEN THE OBJECT IS AT A FINITE DISTANCE.

Methods of elimination of chromatic aberration

- **Achromatic Doublet**

  A combination of convex lens made up of by crown glass and concave lens made up of by flint glass.

- **Arranging two converging lenses coaxially**

  Two convex lenses made up of by same material arranged coaxially at a distance of

$$d = \frac{f_1+f_2}{2}$$

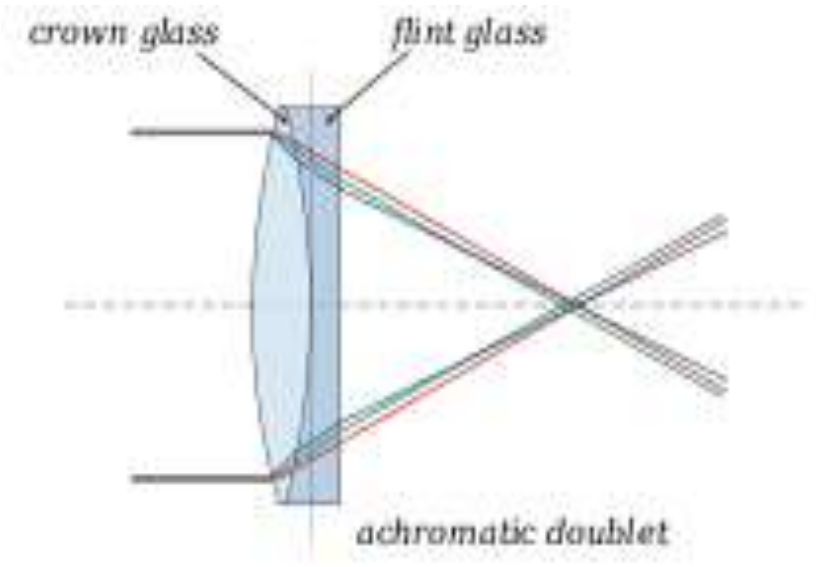
ACHROMATISM: **The method of minimisation/removal of chromatic aberration by combining lenses is called as "ACHROMATISM".**

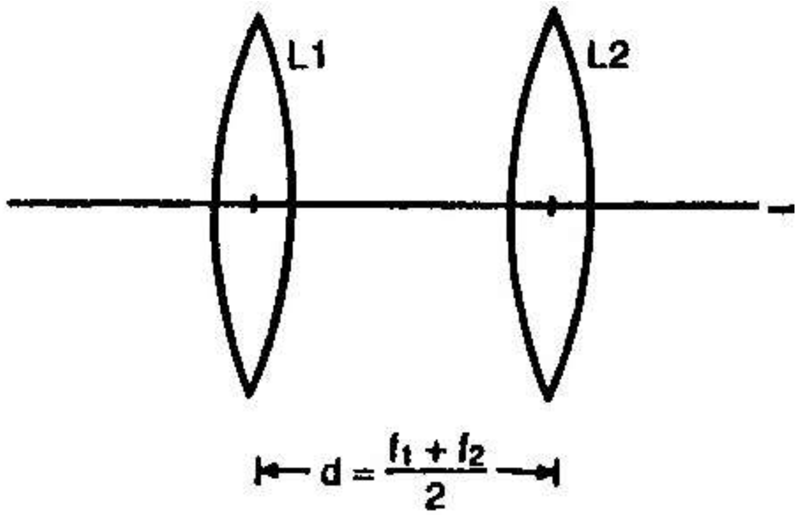
for Convex Lens Chromatic aberration (f_r - f_v) is "+" ve,
for Concave Lens Chromatic aberration(f_r - f_v) is " --"ve.
So the combination of Convex and Concave Lenssystem can be minimized/removed Chromatic aberration.

Achromatic doublet: **A type of lens system that we used to remove/**minimize **the Chromatic aberration is called as "Achromatic doublet"**

A combination of convex lens made up of by crown glass and concave lens made up of by flint glass is act as a Achromatic doublet.

ACHROMATIC DOUBLET





Two lenses are contact each other

Two lenses are seperated by a distance

Condition for Achromatism when two lenses are in contact.

Combination of two or more lenses such that there will not be a chromatic aberration is known as achromatic lens and the condition is called as Achromatism.

Mathematically for Achromatism $df = f_r - f_v = 0$

Now let us take two lenses having focal lengths f_1&f_2 . Then the equivalent focal length of combination will be $1/F = 1/f_1 + 1/f_2$

Diffrentiating this equation we have $-dF/F^2 = -df_1/f_1^2 - df_2/f_2^2$

But we have $-df_1/f_1 = \omega_1$ and $-df_2/f_2 = \omega_2$ Where ω_1 and ω_2 are the dispersive power of 1st and 2nd Lenses.

$$-dF/F^2 = \omega_1/f_1 + \omega_2/f_2$$

For achromatism $dF=0 \Rightarrow \omega_1/f_1 + \omega_2/f_2 = 0$

$$\omega_1/f_1 = -\omega_2/f_2 \Rightarrow f_1/f_2 = -\omega_1/\omega_2$$

The negative sign here indicates combination of lense must be of one convex and another concave,if The lenses are made same material then $\omega_1 = \omega_2 = \omega$,$f_1/f_2 = -1 \Rightarrow f_1 = -f_2$

Condition for Achromatism when two lenses are seperated by a distance.

Mathmatically for achromatism $df = f_r - f_v = 0$

Now let us take two lenses having focal length f_1&$f_2$and are separated by a distance x. Then the equivalent focal length of combination will be $1/f=1/f_1 + 1/f_2 - x/(f_1*f_2)$

Diffrentiating this equation we have

$$-df/f^2 = -df_1/f_1^2 - df_2/f_2^2 - x(-df_1/f_1^2f_2 - df_2/f_2^2f_1)$$

But we have $-df_1/f_1=\omega_1$ and $-df_2/f_2 = \omega_2$

$$-df/f^2 = -df_1/f_1^2 - df_2/f_2^2 - x (\omega_1/f_1f_2 + \omega_2/f_2f_1)$$

For achromatism df = 0 => $\omega_1/f_1 + \omega_2/f_2 = x(\omega_1/f_1f_2 + \omega_2/f_2f_1)$

$$1/f_1f_2 (\omega_1 f_2 +\omega_2f_1) = x/f_1f_2 (\omega_1 + \omega_2)$$

=> $x = (\omega_1 f_2 +\omega_2f_1)/(\omega_1 + \omega_2)$,if $\omega_1 = \omega_2=\omega$ then

$x = (f_2 +f_1)/2$ This is the Condition for Achromatism when two lenses are seperated by a distance(x).

MONOCHROMATIC ABERRATIONS

- The deviations in the actual size, shape and position in the image of an object formed by a single lens, when the object is illuminated by single wavelength light.

TYPES OF MONO CHROMATIC ABERRATION

- Spherical Aberrations.
- Astigmatism.
- Coma.
- Distortion.
- Curvature.

SPHERICAL ABERRATION

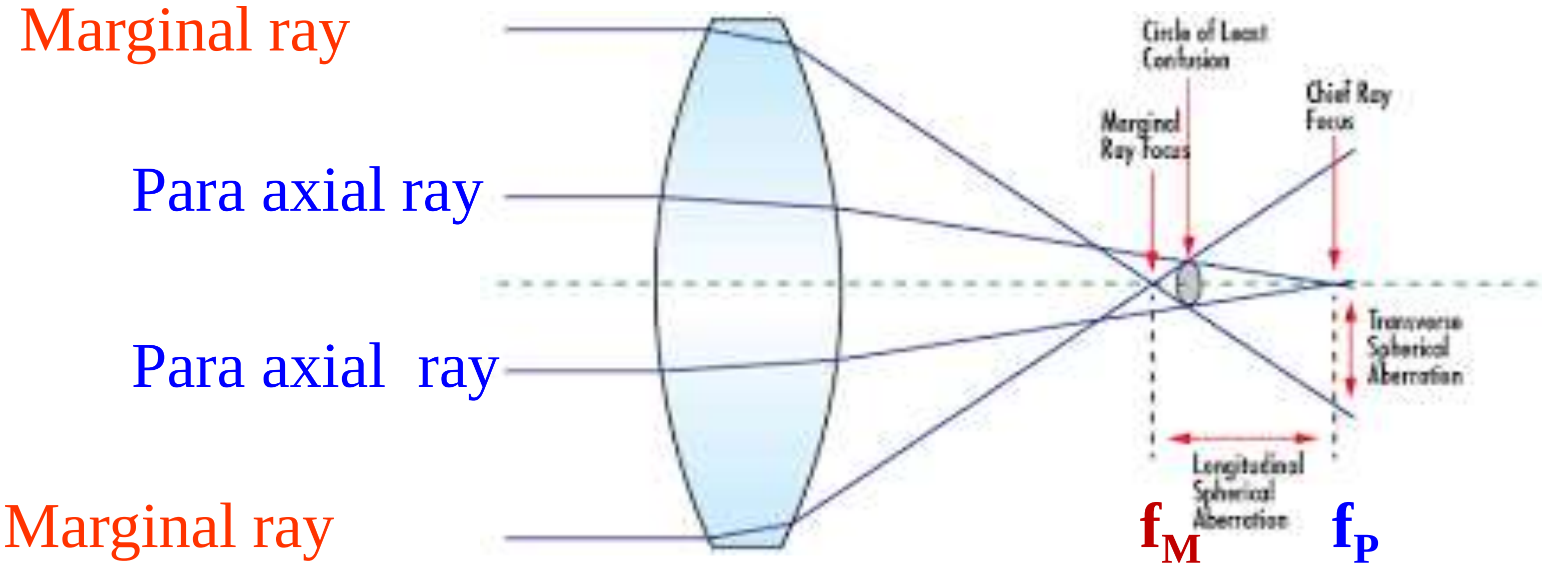
The failure or inability of the lens to form a point image of an axial point object.

TYPES :

Longitudinal : the formation of image in different positions along the axis.

Lateral : the formation of image in different sizes perpendicular to the axis.

VISUAL OF LONGITUDINAL SPHERICAL ABERRATION



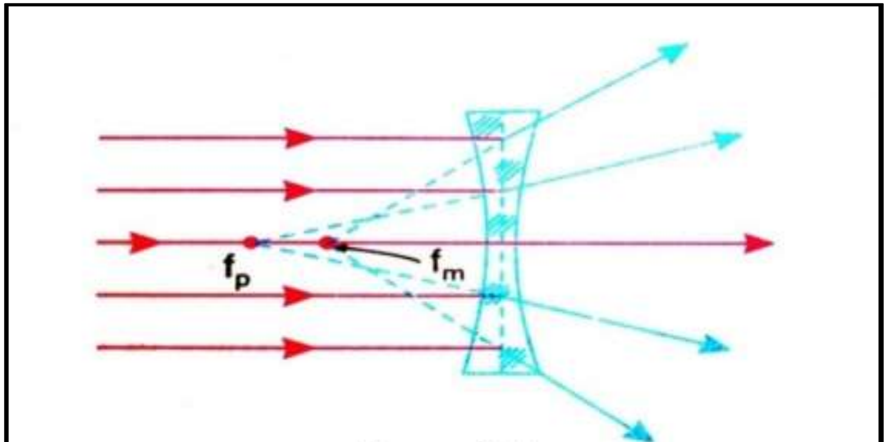
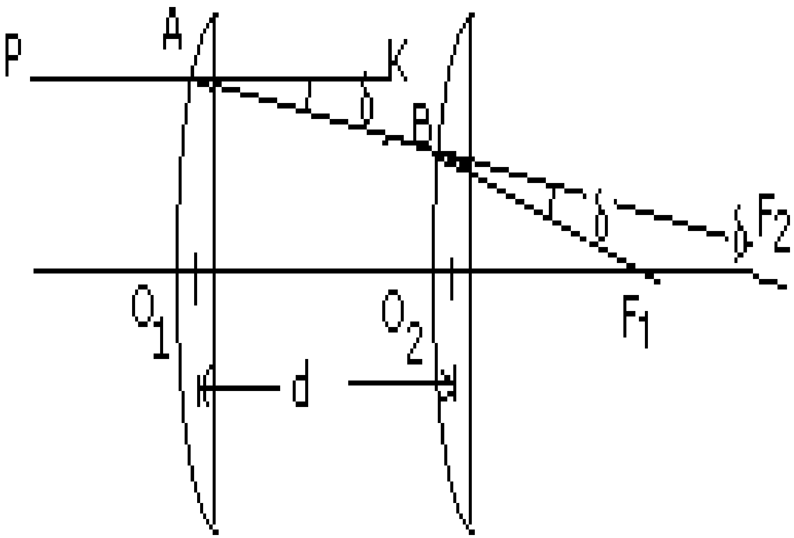


Figure : (11)

METHODS OF ELIMINATION OF SPHERICAL ABERRATION

- By means of stops.
- By using two suitable lenses in contact.
- By using crossed lens.
- By using two plano – convex lenses separated by a distance.

By using two plano – convex lenses separated by a distance.



With reference to figure (4), we can write,
$\angle BAK = \angle BF_2O_2 = \delta$, Also, $\angle F_1 BF_2 = \angle BF_2 F_1 = \delta$, So that $F_1F_2 = F_1B = F_1O_2$
Or $O_2F_1 = ½ O_2F_2$

Since F_2 is the virtual object of the real image F_1 and using the lens formula for the second lens,we can write the equation,

$$1/v - 1/u = 1/f_2 \text{ -------}\rightarrow (1)$$

In this equation $u = f_1 - d$ & $v = (f_1 - d)/2$ Substituting for 'u' and 'v'

and simplifying, we get, $d = f_1 - f_2$

ASTIGMATISM

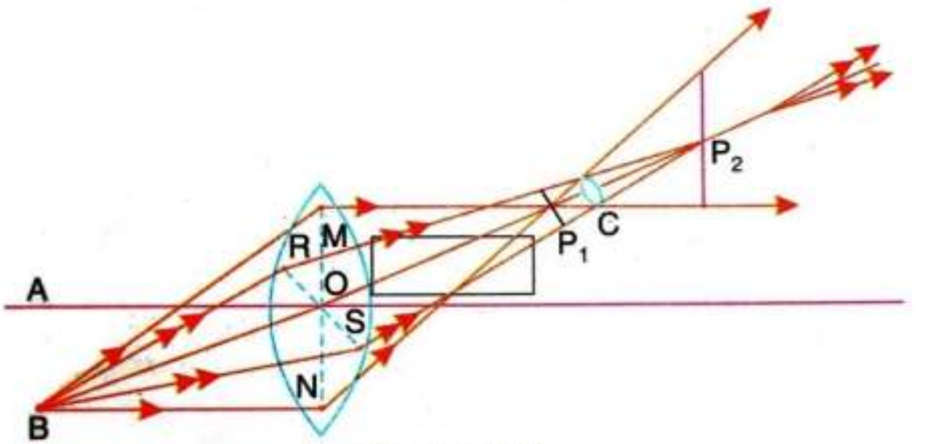


Figure : (21)

When a point object is situated far of the axis of a lens, the image formed by the lens is not in a perfect focus.

It consists two mutually perpendicular lines separated by finite distance.

The tangential (vertical) plane BMN comes to focus at point P_1 nearer the lens and the cone of rays refracted through the sagitial (horizontal) plane BRS comes to focus at the point P_2 away from the lens, the cone of rays refracted through the meridian(vertical) plane.

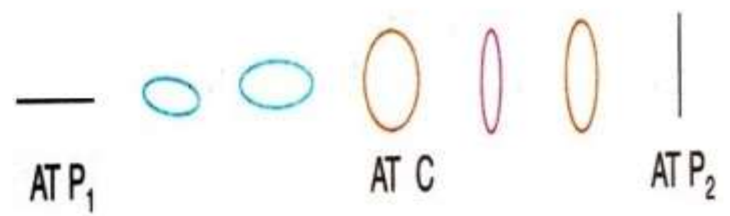


Figure : (22)

Coma:

When object point is not situated on the axis then aberration produced by the lens is called Coma.

Theory: In the case of Spherical aberration, the image is a circle of varying diameter along the axis and in the case of Comatic aberration the image is comet shaped and hence the name is coma.

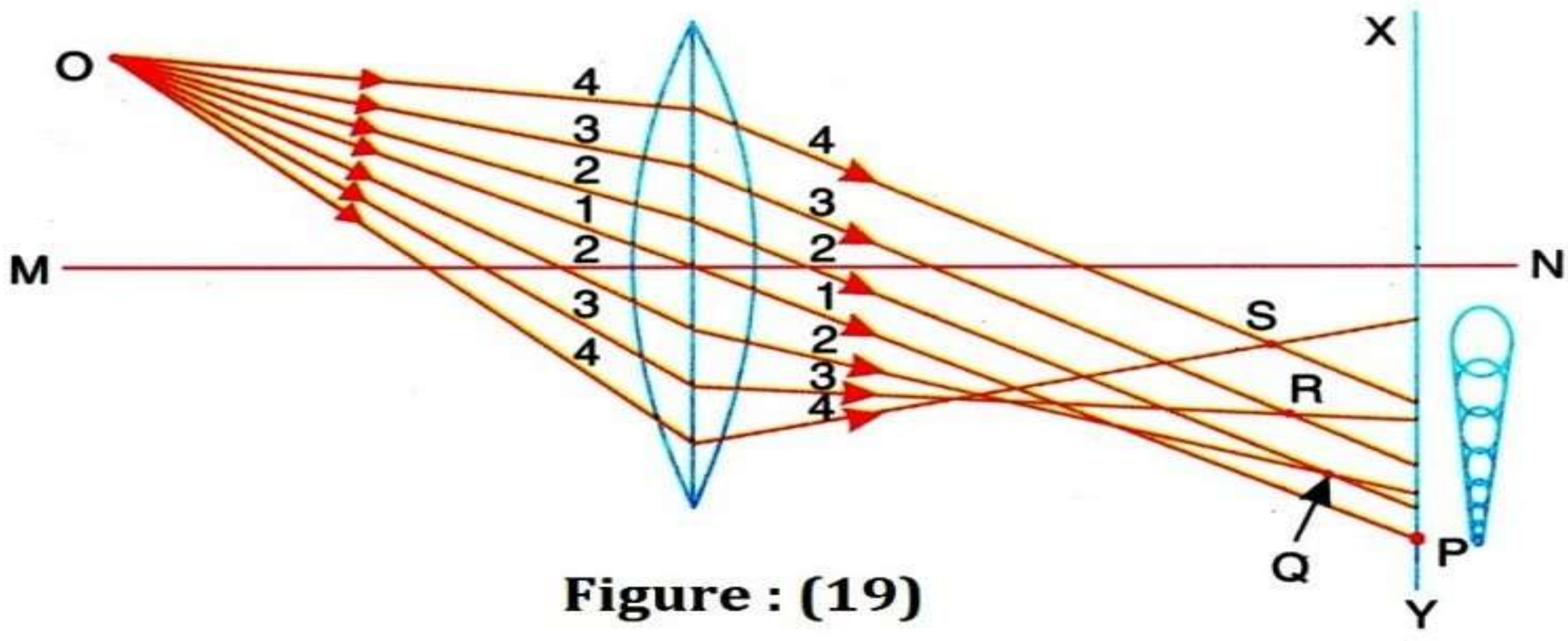


Figure : (19)

- **Coma is the result of varying magnification for rays refracted through different zones of the lens.**

The rays of light getting refracted through the centre of the lens(ray 1) meet the screen XY at the point P. rays 2.2, 3.3, etc getting refracted through the outer zones of the lens come to focus at points Q,R,S, etc. , nearer the lens and overlapping circular patches of gradually increasing diameter are formed on the screen. The resultant image of the point is comet-shaped as shown on the right side of the figure.

Methods of reduction:

- **The coma can be corrected by properly choosing the radii of curvature of the lens surface. A lens corrected for coma will not be free from spherical aberration and the one corrected for spherical aberration will not be free from coma.**
- **Use of a stop or a diaphragm at the proper position eliminates coma.**
- **According to abbe, a German optician, coma can be eliminated if a lens satisfies the Abbe's sine condition viz.**

$$\mu_1 y_1 \sin\theta_1 = \mu_2 y_2 \sin\theta_2$$

Curvature of the field

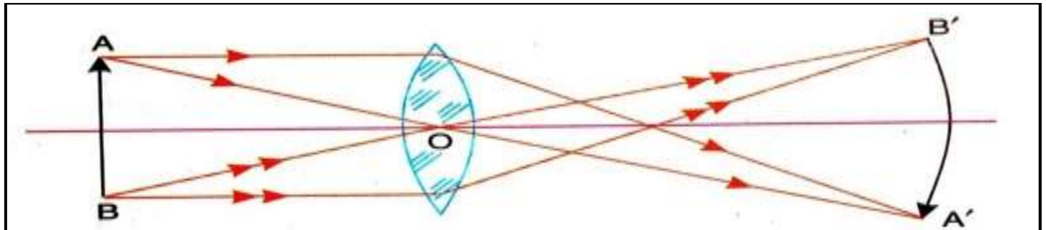


Figure : (24)

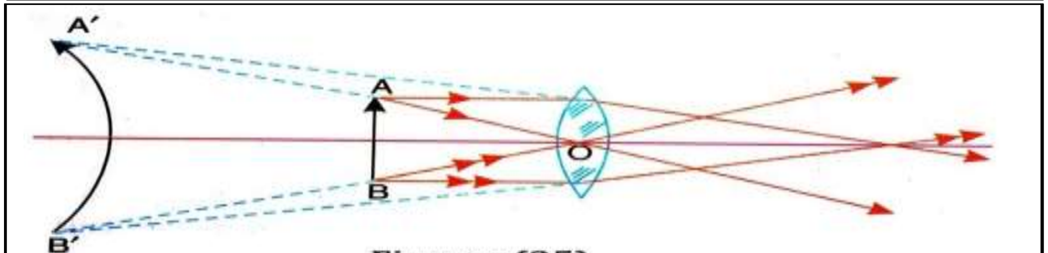


Figure : (25)

The image of an extended object due to a single lens is not a flat one but it will be a curved surface. The central portion of the image nearer the axis is in focus but the outer regions of the image away from the axis are blurred. The paraxial focal length is greater than the marginal focal length. This defect is called the curvature of the field.

For a system of thin lenses, the curvature of the final image can, theoretically, be given by the expression

$$1/R = \sum 1/\mu_n f_n$$

Where R is the radius of the final image, m_n and f_n are the refractive index and focal length of the n^{th} lens. For the image to be flat, R must be infinity

$$1/R = \sum 1/\mu_n f_n = 1/\infty = 0$$

Correspondingly, the condition for two lenses placed in air, reduces to $1/\mu_1 f_1 + 1/\mu_2 f_2 = 0$

This is known as Petzwal's condition for no curvature. This condition holds good whether the lenses are separated by a distance or placed in contact. As the refractive indices m_1 and m_2 are positive, the above condition will be satisfied if the lenses are of opposite sign. If one of the lenses is convex the other must be concave.

DISTORTION: The variation in the magnification produced by a lens for different axial distances produce an aberration is called distortion. There are two types of distortion: (1) pin-cushion distortion and (2) barrel-shaped distortion.

In pin-cushion distortion, the magnification increases with increasing axial distance and the image of an object is shown in figure (29 b).

In barrel-shaped distortion, the magnification decreases with increasing the axial distance and the image of an object is shown in figure (29 c).

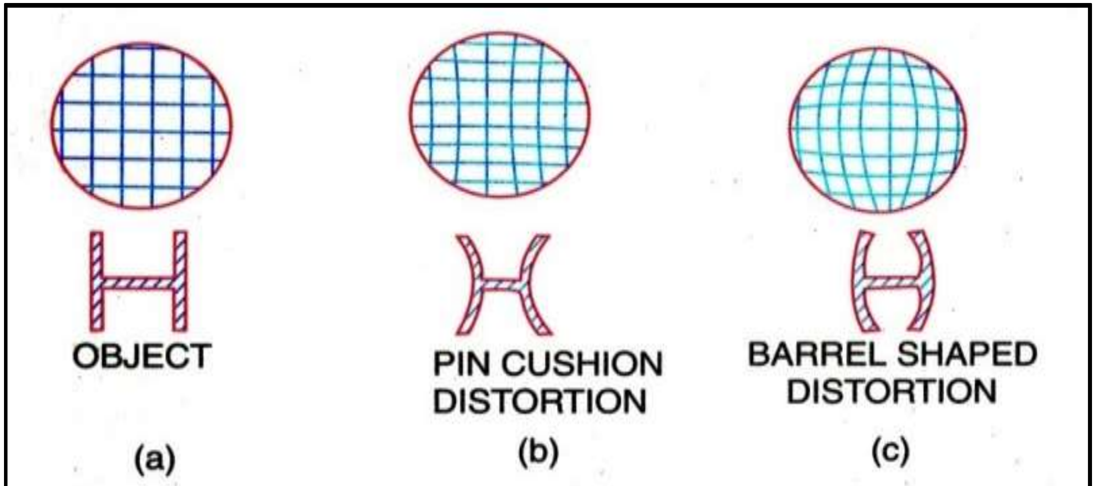


Figure : (29)

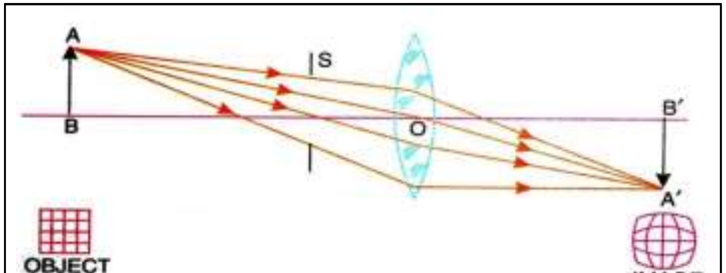


Figure : (30)

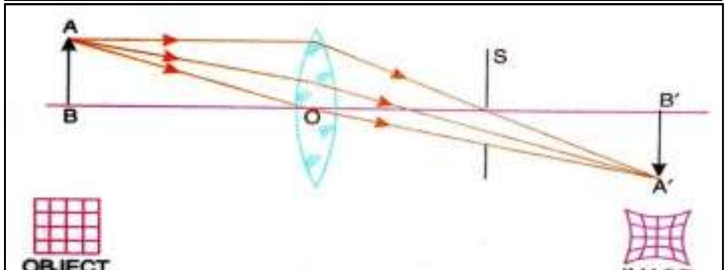


Figure : (31)

To eliminate distortion a stop is placed between two symmetrical lenses, so that the pin-cushion distortion produced by the first lens is compensated by the barrel-shaped distortion produced by the second lens. see figure (32). Projection and camera lenses are constructed in this way.

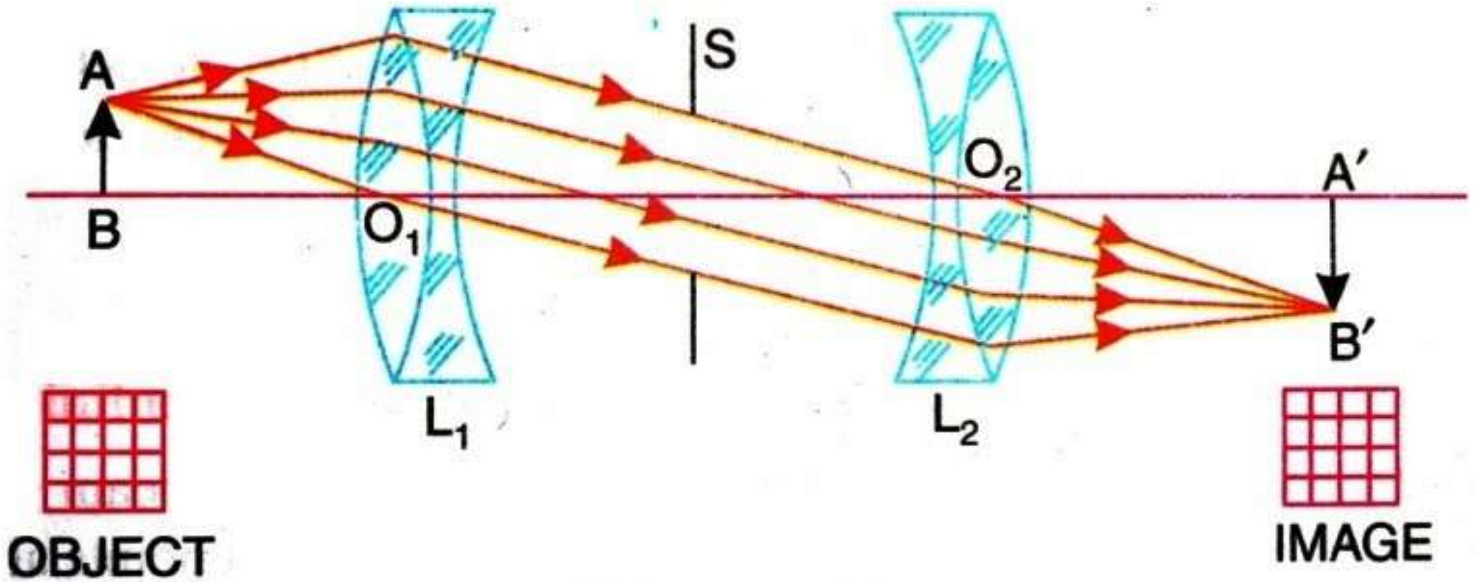


Figure : (32)

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